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SIMPLE WAVE EQUATIONS OF ONE-DIMENSIONAL MOTION OF A GAS – DUST MIXTURE

V. V. Zholobov and L. G. Zholobova

UDC 532.529

§1. The equations of one-dimensional nonstationary motion of a gas–dust mixture [1] can be written in the following form:

$$\frac{\partial}{\partial t} v_1 - \frac{\partial}{\partial \xi} u_1 = 0, \tag{1.1}$$

$$v_1 \frac{\partial}{\partial t} v_2 + (u_2 - u_1) \frac{\partial}{\partial \xi} v_2 - v_2 \frac{\partial}{\partial \xi} u_2 = 0, \quad \frac{\partial}{\partial t} u_1 + \frac{\partial}{\partial \xi} p = -v_1 f_{12},$$

$$v_1 \frac{\partial}{\partial t} u_2 + (u_2 - u_1) \frac{\partial}{\partial \xi} u_2 = v_2 v_1 f_{12},$$

$$v_1 \frac{\partial}{\partial t} p + \gamma p \frac{\partial}{\partial t} v_1 = (\gamma - 1) v_1 f_{12} (u_1 - u_2) - \beta_1 (T_1 - T_2),$$

$$v_1 \frac{\partial}{\partial t} T_2 + (u_2 - u_1) \frac{\partial}{\partial \xi} T_2 = \beta_2 v_1 (T_1 - T_2),$$

$$v_1 d\xi = dx - u_1 dt,$$

where u_i, v_i, and T_i are the velocities, specific volumes, and temperatures of the phases (the subscript 1 refers to the parameters of the gas); p is the pressure; f_{12} is the volumetric force due to the interaction between the gas and the particles as a result of frictional forces; and γ is the ratio of the specific heat capacities of the gas. The coefficients β_i have the form

$$\beta_1 = \frac{6(\gamma - 1) v_1}{\rho_2^0 d v_2}, \quad \beta_2 = \frac{6}{\rho_2^0 d c_2} \alpha, \quad \alpha = \frac{\lambda_1}{d} \mathrm{Nu},$$

where ρ_2^0 is the true density of the second phase, d is the diameter of the particles; λ_1 is the thermal conductivity; and Nu is the Nusselt number. The terms reflecting the force interaction and thermal interaction between the phases are expressed in concrete form as follows [2]:

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$$\mathrm{Nu} = 2 + 0.6\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}, \qquad \mathrm{Pr} = c_p\mu_1/\lambda_1,$$

$$\mathrm{Re} = \frac{|u_1 - u_2|\,d}{\mu_1 v_1}, \; f_{12} = \frac{3}{4}\frac{c_d}{d}\frac{(u_1 - u_2)\,|u_1 - u_2|}{\rho_2^0 v_2 v_1},$$

where c_p, μ_1, and c_d are the specific heat capacity at constant pressure, the dynamic viscosity of the gas, and the coefficient of head resistance of spherical particles. Hereafter we shall use the following relation between c_d and the Reynolds criterion, as given in [3]:

$$c_d = \frac{24}{\mathrm{Re}}\left(1 + 0.197\,\mathrm{Re}^{0.63} + 2.6\cdot 10^{-4}\,\mathrm{Re}^{1.38}\right).$$

The system (1.1) is closed by the relations

$$pv_1 = RT_1, \; \rho_2^0 = \mathrm{const}, \; \mu_1 = \mu_1^0 (T_1/T^0)^m,$$

where m is a constant which depends on the nature of the gas.

In the system (1.1) we pass to dimensionless quantities in accordance with the formulas

$$\begin{gathered} \xi^* = \xi v_1^0/l^0, \; p^* = p/\gamma p^0, \; v_i^* = v_i/v_i^0, \; u_i^* = u_i/a^0, \\ T_i^* = T_i/T^0, \; t^* = a^0 t/l^0, \; a^* = a/a^0, \; f_{12}^* = (l^0/a^{02})\,f_{12}, \\ a^{02} = \gamma p^0 v_1^0, \; \beta_1^* = \beta_1 l^0/\gamma R a^0, \; \beta_2^* = l^0\beta_2/a^0, \end{gathered} \tag{1.2}$$

where the superscript 0 indicates the initial parameters of the mixture; l^0 is the characteristic dimension. The form of the system (1.1) does not change, and hereafter we shall omit the asterisk.

Assuming that all the desired functions depend on one level function [4], and using the velocity of the gas as this level function, from the first and next-to-the-last equations of the system (1.1) we find

$$\begin{gathered} \frac{\partial}{\partial t} u_1 = \frac{(\gamma - 1) f_{12} v_1 (u_1 - u_2) - \beta_1 (T_1 - T_2)}{v_1 \dfrac{dp}{du_1} + \gamma p \dfrac{dv_1}{du_1}}, \\ \frac{\partial u_1}{\partial \xi} = \frac{dv_1}{du_1}\frac{\partial u_1}{\partial t}. \end{gathered} \tag{1.3}$$

Equating the mixed derivatives, after some simple transformations we obtain the integrability condition for (1.3) in the form

$$dv_1/du_1 = \delta = \mathrm{const}. \tag{1.4}$$

Substituting (1.3) into the remaining equations of the system (1.1), we obtain the following system of ordinary differential equations:

$$v_1 \frac{dv_2}{du_1} + (u_2 - u_1)\frac{dv_1}{du_1}\frac{dv_2}{du_1} - v_2 \frac{du_2}{du_1}\frac{dv_1}{du_1} = 0; \tag{1.5}$$

$$v_2 + v_1 \frac{du_2}{du_1}\left[(u_2 - u_1)\frac{du_2}{du_1} + v_2 \frac{dp}{du_1}\right] = 0; \tag{1.6}$$

$$\frac{du_2}{du_1} = \frac{v_2 f_{12}}{\beta_2 (T_1 - T_2)}\frac{dT_2}{du_1}; \tag{1.7}$$

$$\left[v_1 \frac{dp}{du_1} + \gamma p \frac{dv_1}{du_1}\right] \frac{\beta_2 v_1 (T_1 - T_2)}{\left[v_1 + (u_2 - u_1)\dfrac{dv_1}{du_1}\right]} = \{(\gamma - 1) f_{12} (u_1 - u_2) v_1 - \beta_1 (T_1 - T_2)\}\frac{dT_2}{du_1}. \tag{1.8}$$

It can be seen that the system (1.4)-(1.8) has the integrals

$$v_1 = \delta u_1 + C_1, \; v_2 = C_2 (\delta u_2 + C_1), \; p = C_3 - \frac{1}{\delta}\left\{u_1 + \frac{1}{C_2} u_2\right\}, \tag{1.9}$$

where C_1, C_2, and C_3 are arbitrary constants. Taking account of (1.9), we can reduce the system (1.4)-(1.8) to two ordinary differential equations of first order:

$$\begin{gathered} \frac{du_2}{du_1} = \frac{v_2 f_{12}}{\beta_2 (T_1 - T_2)}\frac{dT_2}{du_1}, \\ \frac{dT_2}{du_1} = \frac{\left(\delta\gamma p - \frac{1}{\delta} v_1\right)(T_1 - T_2)\,\beta_2 v_1 (T_1 - T_2)}{(\delta u_2 + C_1)\,\{f_{12} v_1 [(\gamma - 1)(u_1 - u_2) + v_1/\delta v_2] - \beta_1 (T_1 - T_2)\}}. \end{gathered} \tag{1.10}$$

The level function is determined from (1.3), taking account of (1.4):

$$t+\delta\xi=\int_0^{u_1}\frac{\left(v_1\frac{dp}{du_1}+\gamma p\frac{dv_1}{du_1}\right)du_1}{(\gamma-1)f_{12}v_1(u_1-u_2)-\beta_1(T_1-T_2)}+C_4. \qquad \mathbf{(1.11)}$$

Thus, the simple wave described the self-similar motion determined by the transfer group [5]. For numerical calculations, it is preferable to use the variable u_1 instead of $(t+\delta\xi)$, since the range of integration becomes finite.

§2. We shall make use of the resulting relations to describe the motion occurring in front of a piston which moves according to a specified law into a mixture which is initially at rest. On the curve $\xi=-(1/\delta)t$ we specify the following conditions:

$$u_1=0,\ u_2=0,\ v_1=1,\ v_2=v_2^0,\ T_1=1,\ T_2=1,\ p=1/\gamma. \qquad (2.1)$$

We assume that the collision of the particles with the piston surface is absolutely inelastic and that the particles hitting the piston surface form a layer in a dense-packed state (a film). Using this model of the interaction between the particles and the piston surface leads to the following boundary conditions for the surface of the layer:

$$\xi=0,\ x=x_++\Delta,\ u_1=u_++d\Delta/dt,$$

where x_+ and u_+ are the coordinate and velocity of the piston, respectively; Δ is the thickness of the layer (the film), determined from the differential equation

$$\frac{d\Delta}{dt}=\frac{u_1-u_2}{v_2\rho_2^0\varepsilon_2^*}\quad(\Delta=0,\ t=0) \qquad (2.2)$$

(ε_2^* is the volumetric content of particles in the dense-packed state). The pressure in the film is distributed according to the law

$$p_+=p+\Delta p-\varepsilon_2^*\rho_2^0(\delta-x)\frac{\partial}{\partial t}u_+,$$

where Δp is the local increase in pressure, by which we mean the pressure exerted by a particle on the surface of the layer upon collision [6], calculated according to the formula

$$\Delta p=\eta(u_1-u_2)^2/v_2.$$

Taking account of (2.1), we find from (1.9) that

$$C_1=1,\ C_2=v_2^0,\ C_3=1/\gamma,\ C_4=0. \qquad (2.3)$$

From the conditions of kinematic consistency it follows that $\delta=-1$.

Thus, the solution of Eqs. (1.10) and (2.2) with the initial conditions $u_2(0)=0$, $T_2(0)=1$, $\Delta(0)=0$ and the relations (1.9), (1.11), and (2.3) describe the flow of a gas–dust mixture occurring in front of a piston moving according to the law

$$t=\int_0^{u_1(u_+)}\frac{\left(v_1\frac{dp}{du_1}+\gamma p\frac{dv_1}{du_1}\right)du_1}{(\gamma-1)f_{12}v_1(u_1-u_2)-\beta_1(T_1-T_2)}, \qquad (2.4)$$

$$u_1=u_++\frac{u_1-u_2}{v_2\rho_2^0\varepsilon_2^*}$$

into a mixture which is initially at rest.

We mark the parameters of the flow taking place in an equilibrium mixture in front of a piston moving inward according to the law (2.4) with a subscript asterisk. The equilibrium flow of a dusty gas is described by the classical equations of gasdynamics, but γ – the ratio of the specific heat capacities – is replaced by some effective value

$$\gamma_*=\frac{c_p+c_2/v_2^0}{c_v+c_2/v_2^0},$$

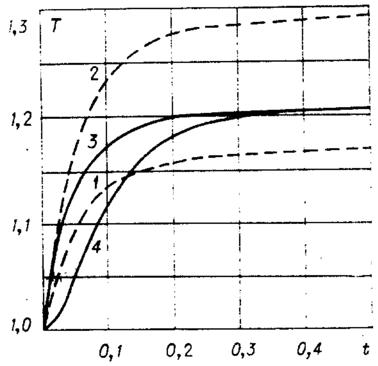

Fig. 1

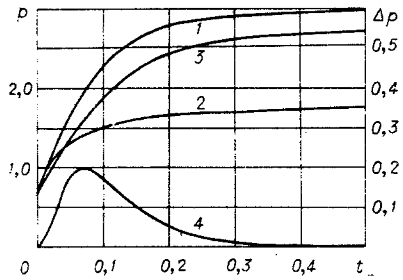

Fig. 2

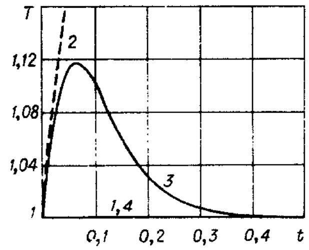

Fig. 3

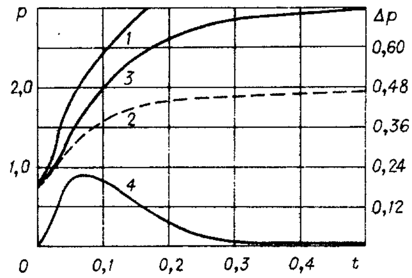

Fig. 4

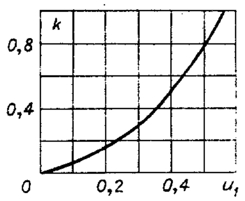

Fig. 5

and the density is taken to mean the total density of the mixture [7]. Making use of a solution of the simple-wave type [4], we can satisfy the boundary condition on the piston and the initial conditions (2.1) on the curve $\xi = t/a$. As a result, we obtain

$$p_* = \frac{1}{\gamma}\left\{1 + \frac{\gamma_* - 1}{2} a u_*\right\}^{2\gamma_*/(\gamma_* - 1)}, \quad (2.5)$$

$$v_* = \frac{v_2^0}{1+v_2^0}\left\{1 + \frac{\gamma_* - 1}{2} a u_*\right\}^{2/(1-\gamma_*)},$$

$$t - a\left\{1 + \frac{\gamma_* - 1}{2} a u_*\right\}^{(\gamma_* + 1)/(1-\gamma_*)} \xi = \Phi(u_*), \; a = \sqrt{\frac{\gamma(1+v_2^0)}{\gamma_* v_2^0}}.$$

The function $\Phi(u_*)$ is determined from (2.4) if we set $t = \Phi(u_*)$. In the case of a clean gas, in the choice of the dimensionless quantities in the form (1.2), the solution of the fundamental system of equations satisfying the boundary condition on the piston and the initial conditions (2.1) on the curve $\xi = t$ has the form

$$p = \frac{1}{\gamma}\left\{1 + \frac{\gamma-1}{2} u\right\}^{2\gamma/(\gamma-1)}, \; v = \left\{1 + \frac{\gamma-1}{2} u\right\}^{2/(1-\gamma)}, \quad (2.6)$$

$$t - \left\{1 + \frac{\gamma-1}{2} u\right\}^{(\gamma+1)/(1-\gamma)} \xi = \Phi(u).$$

Numerical calculations were carried out for $\Delta \equiv 0$ and the following values of the thermodynamic parameters of the phases: $c_p = 1047.5$ J/kg·deg; $c_2 = 800$ J/kg·deg; $d = 10^{-5}$ m; $\lambda_1 = 0.05$ J/m·sec·deg; $\mu_1^0 = 1.86 \cdot 10^{-5}$ N·sec/m²; $R = 287.29$ J/kg·deg; $l^0 = 1$ m; $T_1^0 = T_2^0 = 300$°K; $p = 10^{-5}$ N/m²; $\gamma = 1.4$; $m = 0.5$; and $v_1^0/v_2^0 = 2$.

Figures 1 and 2 show the distributions of temperatures and pressures along the line of the piston, respectively. Curves 1 and 2 will hereafter correspond to the parameters calculated by formulas (2.5) and (2.6), curve 3 to the parameters of the gas, and curve 4 to the parameters of the particles. In the case under consideration, formulas (2.5) yield an error of not more than 4% in the determination of the mixture temperature. It should be noted that the solution (2.5), (2.6) is valid to the time when a shock wave is formed, i.e., in the region bounded by the straight lines $\xi = 0$, $\xi = t/a$ (or $\xi = t$) and the corresponding characteristic passing through the point of formation of the shock wave.

§3. If there is no dynamic disequilibrium ($u_1 = u_2$), the conditions (2.1) are given along the curve $\xi = \sqrt{1 + 1/v_2^0}\, t$. In this case we obtain the results known from [5]:

$$p = \frac{1}{\gamma} + b u_1, \; v_1 = 1 - b u_1, \; v_2 = v_2^0 v_1,$$

$$T_2 = 1 + \frac{b^2(\gamma+1)}{2\alpha} u_1^2, \quad \alpha = \frac{c_2(\gamma-1)}{\gamma R v_2^0}, \; b = \sqrt{1 + \frac{1}{v_2^0}}, \; \Delta = 0,$$

$$u_1 = \frac{2\alpha(\gamma-1)b}{\gamma(1+2\alpha)+1}\left\{1 - \exp\left[-\frac{1+\gamma(1+2\alpha)}{2(\gamma+1)} \beta_2 b (t - b\xi)\right]\right\}.$$

In the limiting case $c_2 = \infty$ ($\gamma_* = 1$, $T_2 = 1$), instead of (2.5) we have

$$p_* = (1/\gamma) \exp a u_*, \quad v_* = (1/b^2) \exp(-a u_*),$$

$$t - a\xi \exp(-a u_*) = \Phi(u_*).$$

The distributions of the gas temperature and the pressure in this case are shown in Figs. 3 and 4.

If there is no thermal disequilibrium between the phases ($T_2 = T_1$), then instead of the first equation of (1.10) we have

$$\frac{du_2}{du_1} = \frac{\gamma\delta C_2 C_3 - \gamma C_2(u_1 + u_2/C_2) - (u_1 + C_1/\delta) C_2}{(\gamma-1)(u_1 - u_2) + u_1 + C_1/\delta}. \quad (3.1)$$

Taking account of (2.1), we can write the solution of Eq. (3.1) as follows:

$$u_2 = \frac{\{\gamma - [\gamma^2 + (\gamma^2-1)v_2^0] u_1\}}{\gamma^2 + (\gamma^2-1)v_2^0}\left\{-\frac{\gamma}{\gamma+1} + \right.$$

$$\left. + \sqrt{\frac{\gamma^2 + (\gamma^2-1)v_2^0}{(\gamma-1)^2} + \frac{v_2^0(\gamma+1) + 2\gamma w_0 - (\gamma-1)w_2^0}{(\gamma-1)\left[\frac{\gamma^2 + (\gamma^2-1)v_2^0}{\gamma} u_1 - 1\right]^2}}\right\} - \frac{v_2^0(\gamma+1)}{\gamma^2 + (\gamma^2-1)v_2^0},$$

where $w_0 = -v_2^0(\gamma + 1)/\gamma$. In this case, from (1.3), (2.2), (1.11), and (2.4) we obtain the solution of the previously formulated problem in quadratures. Figure 5 shows the graph of the coefficient of slippage of the phases, $k = u_2/u_1$.

In the case $u_2 = 0$, instead of Eqs. (1.6), (1.7), we obtain a single equation:

$$\frac{1 + \frac{dp}{du_1}\frac{dv_1}{du_1}}{v_1\frac{dp}{du_1} + \gamma p \frac{dv_1}{du_1}}\{(\gamma - 1) f_{12} u_1 v_1 - \beta_1 (T_1 - T_2)\} = -f_{12} v_1.$$

The solution of the previous problem reduces to the simultaneous integration of Eq. (2.2) and the equations

$$\frac{dp}{du_1} = 1 + \frac{f_{12}}{(T_1 - T_2)\beta_2}\frac{dT_2}{du_1},$$
$$\frac{dT_2}{du_1} = \frac{\beta_2(\gamma p + u_1 - 1)(T_1 - T_2)(1 - u_1)}{\beta_1(T_1 - T_2) + (1 - u_1)(1 - \gamma u_1)/f_{12}}$$

with the initial conditions

$$p(0) = 1/\gamma, \quad T_2(0) = 1, \quad \Delta(0) = 0.$$

§4. In a two-velocity and two-temperature medium, unlike a one-velocity one-temperature medium, we observe a layering of characteristics and level curves, as is typical of equations with finite right sides. In classical gasdynamics, and also for nonequilibrium flows [8], the point where the shock wave arises is the point of intersection of the nearest characteristics of one family. The position of the first point of intersection on the initial characteristic is determined by the parameters of the gas and the acceleration of the piston at the initial instant of time. The differential equation of the family of characteristics to which the initial characteristic $\xi = -\delta t$ belongs can be written in the form

$$dz/dt = 1 + \sqrt{\gamma p(u_1(z))/v_1(u_1(z))}, \quad z = \xi + \delta t. \tag{4.1}$$

If one of the relaxation processes is not present, the function on the right side of (4.1) satisfies the conditions of Picard's theorem in the region $0 < t$, $0 \le z \le t$. In this region, Eq. (4.1) has a unique solution, which ensures that there will be no shock waves in the region of the flow.

Transition to an inertial system of coordinates, moving with respect to the laboratory system with velocity $U = -1/\delta$, reduces the fundamental system of equations in the case of a simple wave to the stationary form. This enables us to use the result of [9], where it is shown that for $U = -1$ the corresponding flows describe the motion of a mixture of the "continuous shock wave" type. Thus, the flows considered here constitute an example of uninterrupted compression flows, the possibility of whose existence was shown in [8, 10].

The relations (1.9)-(1.11) are of special interest from the viewpoint of confirming numerical methods for the calculation of nonstationary motions of an aerocolloid.

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INTERNAL WAVES GENERATED BY LOCAL DISTURBANCES IN A LINEARLY STRATIFIED LIQUID OF FINITE DEPTH

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UDC 532.593

§1. In order to investigate the internal waves caused by the elongation of an axially symmetric body moving horizontally at constant velocity U in a stratified liquid, we consider the stationary problem of the flow of a uniform stream of heavy liquid of finite depth past a point source and sink of equal magnitude m which are situated below the free surface. The method of solving this problem is analogous to [1], in which we investigated the case of an unbounded liquid.

The source and sink are situated at a depth h below the unperturbed free surface, $y=0$, of the horizontal layer of liquid, $-\infty < x, z < \infty$, $-H \le y \le 0$. The line segment connecting the singularities is of length $2a$ and parallel to the x axis, which coincides with the direction of the velocity vector of the liquid far upstream. In the unperturbed state the distribution of the liquid density has the form

$$\rho_0(y) = \rho_s(1-\alpha y),\ -H \leqslant y \leqslant 0,\ \alpha = \mathrm{const} > 0. \tag{1.1}$$

We assume that for sufficiently deep immersion and weak stratification, the flow past this combination of source and sink is equivalent to the flow past a closed axially symmetric body (analogous to an unbounded homogeneous liquid). The radius R of the midsection, the elongation d of the body, and the velocity U of the fundamental stream uniquely determine the values of a and m [1].

In the linear formulation, making use of the Boussinesq approximation, the equations of motion have the form

$$\partial u/\partial x + \partial v/\partial y + \partial w/\partial z = m[\delta(x+a) - \delta(x-a)]\delta(y+h)\delta(z), \tag{1.2}$$
$$\rho_s U\partial u/\partial x = -\partial p/\partial x,\ \ \rho_s U\partial v/\partial x = -\partial p/\partial y - g\rho,\ \ \rho_s U\partial w/\partial x = -\partial p\partial z,$$
$$U\partial\rho/\partial x - \alpha\rho_s v = 0$$

with the boundary conditions

$$v=0,\ y=0,\ y=-H,\ u,\ v,\ w,\ p,\ \rho\to 0,\ x^2+z^2\to\infty,$$

where u, v, w, p, and ρ are the perturbations of the components of the velocity vector in the directions of the x, y, and z axes, the pressure, and the density which are caused by the presence of the singularities in the originally unperturbed flow; g is the acceleration of gravity; and δ is the Dirac delta function.

The free surface is replaced by a rigid "lid", since for sufficiently deep immersion the surface waves are negligibly small and the internal waves, for weak stratification, cause practically no distortion in the shape of the free surface [1, 2].

The function $\eta(x, y, z)$, determining the vertical deviation of a liquid particle from its unperturbed state, satisfies the linearized condition $\partial\eta/\partial x = v/U$.

Introducing the dimensionless variables $(x_*, y_*, z_*, h_*, H_*, \eta_*, a_*) = (1/R)(x, y, z, h, H, \eta, a)$, $(u_*, v_*, w_*) = (1/U)(u, v, w)$, and $m_* = m/UR^2$, we reduce Eq. (1.2) to a single equation for the function v_* (the subscript asterisk will be omitted from now on):

$$\frac{\partial^2}{\partial x^2}\Delta v + S\Delta_2 v = m\frac{\partial}{\partial y}\delta(y+h)\frac{\partial^2}{\partial x^2}[\delta(x+a) - \delta(x-a)]\delta(z),$$

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